LINES

"Find the equation for the line…"

Want?

- a POINT on the line (any point!)
- a DIRECTION vector parallel to line

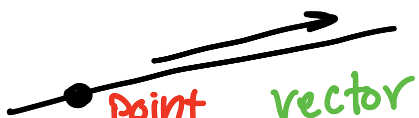
Given?

either the info is given, OR

Find two points

(subtract to get direction)

Step 1: Draw/write



Point vector

$$x = \quad + \quad t$$
$$y = \quad + \quad t$$
$$z = \quad + \quad t$$

Step 2: Given info

PLANES

"Find the equation for the plane…"

Want?

- a POINT on the plane
- a NORMAL perpendicular to plane

Given?

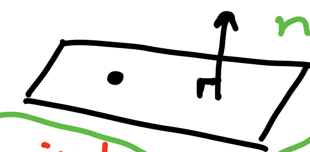
either the info is given, OR

Find two vectors parallel to plane

(usually by first finding 3 pts)

Then do a cross-product

Step 1: Draw, write



normal vector

Point

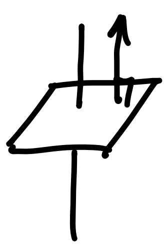
$$(x - \quad) + (y - \quad) + (z - \quad) = 0$$

Step 2: Given info

Tips

Two Parallel Lines ⇒ can use same direction

Two Parallel Planes ⇒ can use same Normal



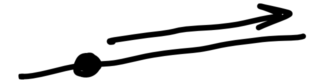
Plane + Line perpendicular ⇒ can use same normal + direction

Winter 2011 Final – 4(a)

Find parametric equations for the line of intersection of the planes $x + 2y - z = 2$ and $3x + 6y - z = 14$.

want

$$\boxed{\begin{aligned} x &= 0 + 6t \\ y &= 3 + -3t \\ z &= 4 + 0t \end{aligned}}$$



Given

① $x + 2y - z = 2$

② $3x + 6y - z = 14$

combine

② − ① $= 2x + 4y + 0 = 12$

$x + 2y = 6$

Pick numbers!

$x = 0$

$2y = 6 \Rightarrow y = 3$

$y = 0$

$x = 6$

Plug in for z

$6 - z = 2$

$z = 4$

A(0, 3, 4)

B(6, 0, 4) → points

$\vec{AB} = \langle 6-0, 0-3, 4-4 \rangle$

$= \langle 6, -3, 0 \rangle$ → direction

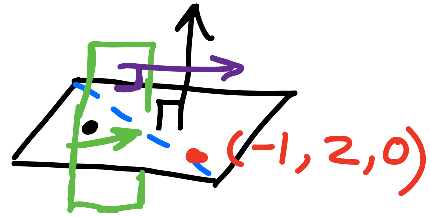
Visual:
https://www.math3d.org/twRnsSoS

contains = on (NOT perpendicular)

Winter 2019 Final – Problem 1(b)

Give the equation of the plane that contains the line:

want



$$x = 4t - 1, y = 2 - 5t, z = t$$

$(x - -1) + (y-2) + (z-0) = 0$

and is perpendicular to the plane

Given $x - y + 8z = 10.$

Point: $(-1, 2, 0)$

contains line: $\begin{cases} x = -1+4t \\ y = 2-5t \\ z = 0+t \end{cases}$ direction: $\langle 4, -5, 1 \rangle$ is parallel to plane

perpendicular to: $x - y + 8z = 10$ $\langle 1, -1, 8 \rangle$ is normal

parallel to plane

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & -5 & 1 \\ 1 & -1 & 8 \end{vmatrix} = (-40-1)i - (32-1)j + (-4--5)k$$

$= \langle -39, -31, 1 \rangle \rightarrow$ normal to plane

$$\boxed{-39(x+1) - 31(y-1) + 1(z-0) = 0}$$

Visual: https://www.math3d.org/ctYo3CgD

Fall 2013 – Exam 1 - Loveless

Consider the line, L_1,

$x = 2 + t, y = 3 - 2t, z = 19 + 7t.$

A second line, L_2, passes through (-3,3,0) and (-1,4,6). Do these lines intersect?

$$L_1 : \begin{cases} x = 2 + t \\ y = 3 - 2t \\ z = 19 + 7t \end{cases}$$

$$L_2 : \begin{cases} x = -3 + 2u \\ y = 3 + u \\ z = 0 + 6u \end{cases}$$

$$\vec{AB} = \langle -1+3, 4-3, 6-0 \rangle$$

$$2 + t = -3 + 2u$$
$$3 - 2t = 3 + u \Rightarrow -2t = u$$
$$19 + 7t = 6u$$

$$2 + t = -3 - 4t$$
$$5t = -5 \Rightarrow t = -1$$
$$u = 2$$

there @ different times but do intersect!

$$19 + 7t = 6u$$
$$12 = 12 \checkmark$$

$$\boxed{\text{yes! } (1, 5, 12)}$$

Visual:
https://www.math3d.org/wydVta8f

$\langle 3, 7, -1 \rangle$

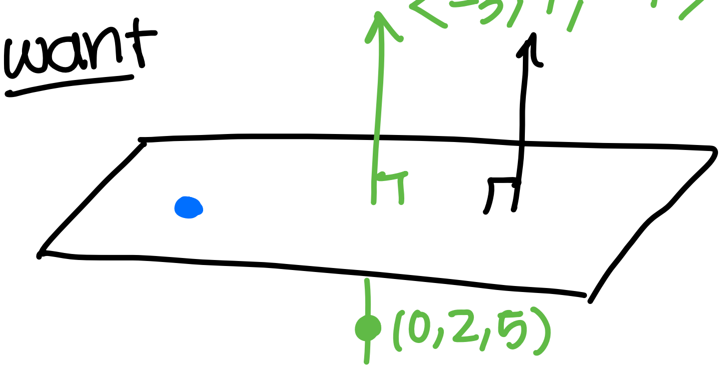
Spring 2013 Exam 1 – Loveless

Consider the plane that contains (1,-1,2) and is orthogonal to

$$x = -3t, y = 2 + 7t, z = 5 - t.$$

Find the equation for this plane. Also give the point of intersection of this plane with the x-axis.

want



$-3(x-1)+7(y+1)-1(z-2)=0$

$$\boxed{-3(x-1)+7(y+1)-1(z-2)=0}$$

Given

orthogonal to:

$x = 0-3t$

$y = 2+7t$

$z = 5-t$

point on x-axis = y is 0, z is 0

$-3(x-1)+7(0+1)-1(0-2)=0$

$-3x+3+7+2=0$

$3x=12$ $\quad x=4$

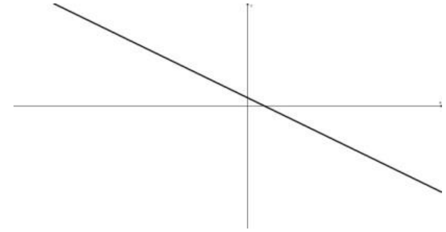
$$\boxed{(4,0,0)}$$

Visual:

https://www.math3d.org/YwmIO6uK

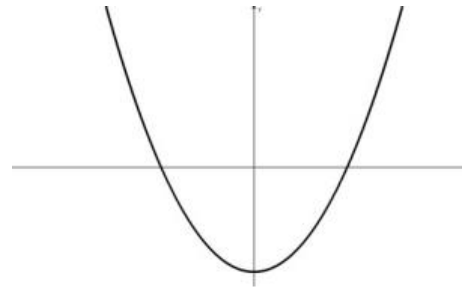
A 2D curve review

Lines: $ax + by = c$



Parabolas: $ax^2 + by = c$ or

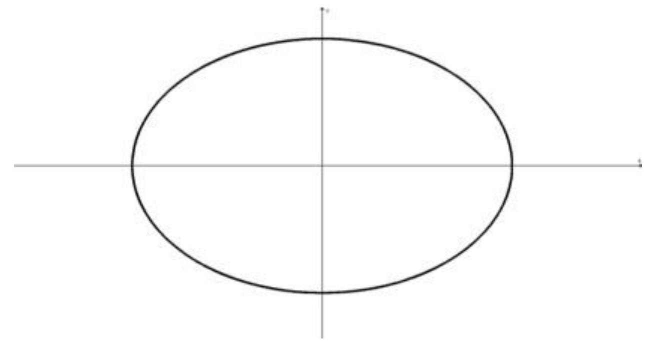
$ax + by^2 = c$



Ellipse: $ax^2 + by^2 = c$ (if $a,b,c > 0$)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

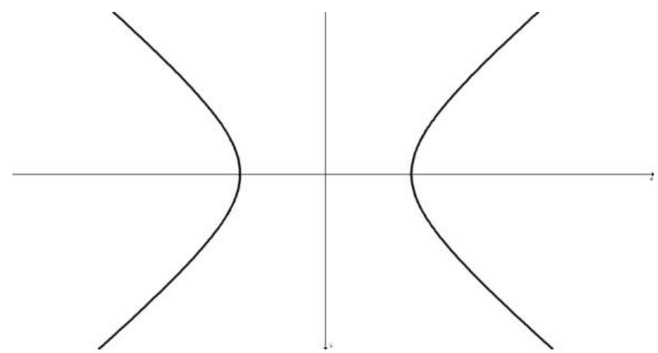
(*Note:* If a = b, then it’s a circle)



Hyperbola: $ax^2 - by^2 = c$ or

$-ax^2 + by^2 = c$ (if $a,b,c > 0$)

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

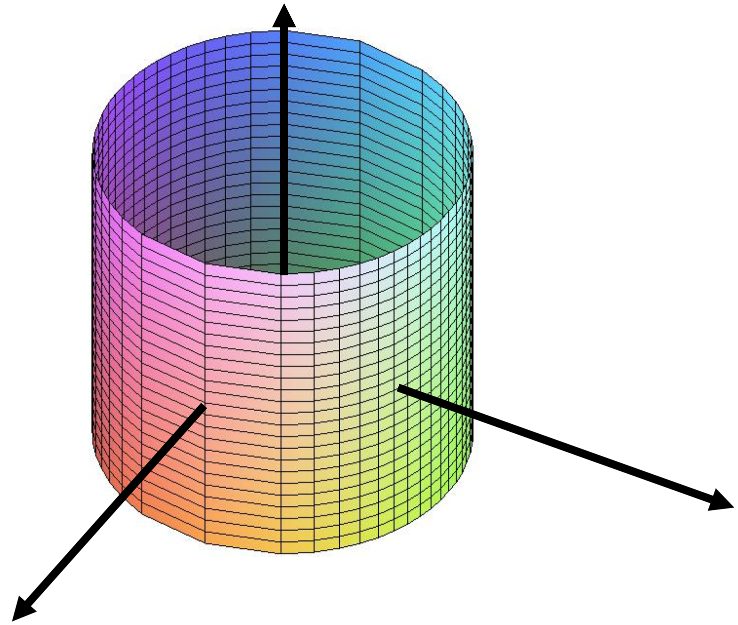


Cylinders: If *one variable is absent*, then the graph is a 2D curve extended into 3D.

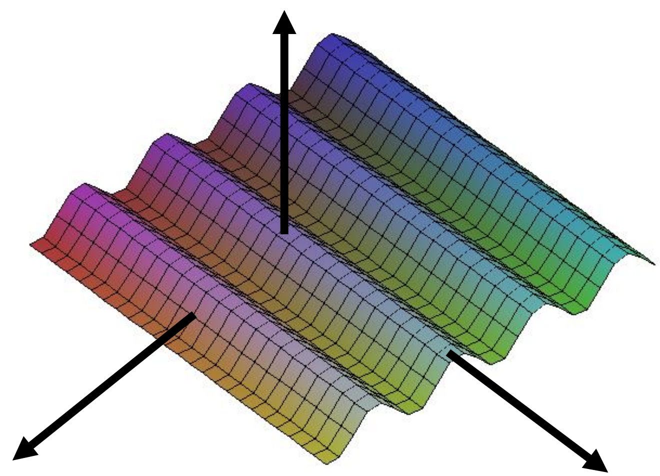
If the 2D shade is called "BLAH", then the 3D shade is called a "BLAH cylinder".

Examples:

(a) $x^2 + y^2 = 1$ in 3D is a **circular cylinder** (*i.e.* a circle extended in the *z*-axis direction).

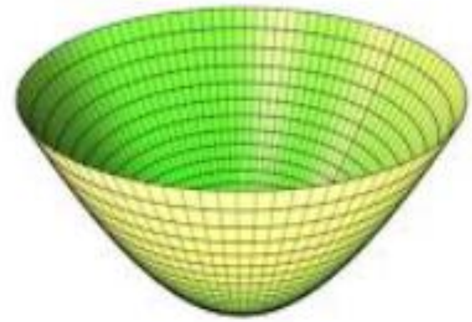


(b) $z = \cos(x)$ in 3D is a **cosine cylinder** (i.e. the cosine function extended in the *y*-axis direction).



Quadric Surfaces: A surface given by an equation involving a sum of first and second powers of x, y, and z is called a *quadric surface*.

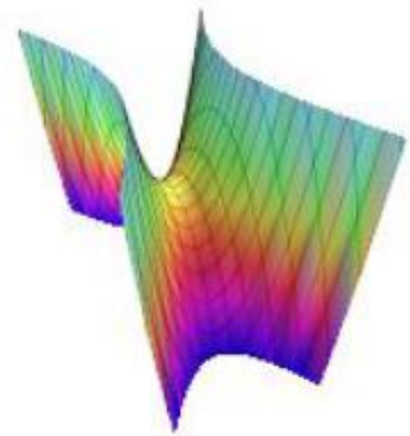
To visualize, we use **traces**.
We fix one variable and look at the resulting 2D picture (i.e. look at one vertical or horizontal slice). If we do several traces in different directions, we start to get an idea about the picture.



Elliptical/Circular Paraboloid

$$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

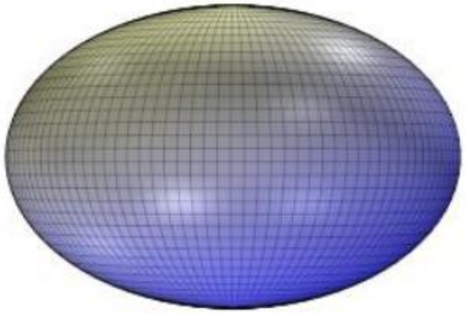
(*ex*: $z = 3x^2 + 5y^2$)



Hyperbolic Paraboloid

$$\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$

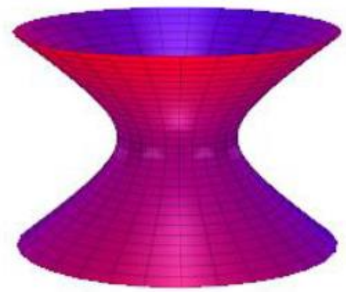
(*ex*: $y = 2x^2 - 5z^2$)



Ellipsoid/Sphere

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

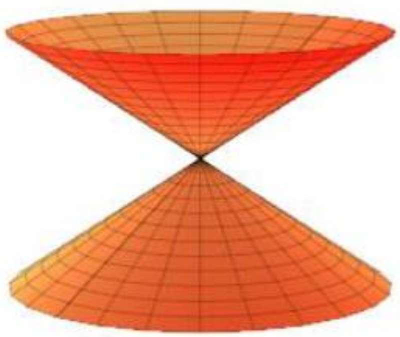
(*ex*: $3x^2 + 5y^2 + z^2 = 3$)



Hyperboloid of One Sheet

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$$

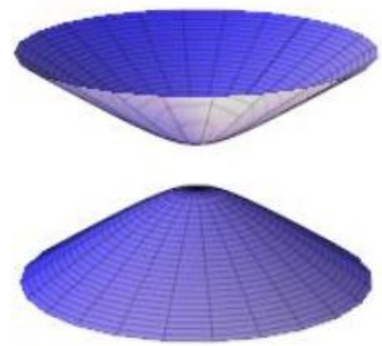
(*ex*: $x^2 - y^2 + z^2 = 10$)



Circular/Elliptical Cone

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$$

(*ex*: $z^2 = x^2 + y^2$)



Hyperboloid of Two Sheets

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$$

(*ex*: $x^2 + y^2 - z^2 = -4$)